Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. If p and q are co-prime numbers, then p^2 and q^2 are **[1]**

a) even
b) coprime
c) not coprime
d) odd

2. If two positive integers ‘a’ and ‘b’ are written as $a = pq^2$ and $b = p^3q^2$, where ‘p’ and ‘q’ are prime numbers, then LCM(a, b) = **[1]**

a) pq
b) p^3q^2
c) p^2q^3
d) p^2q^2

3. The quadratic equation whose roots are 7 + $\sqrt{3}$ and 7 - $\sqrt{3}$ is **[1]**

a) $x^2 + 14x - 46 = 0$
b) $x^2 - 14x + 46 = 0$
c) $x^2 - 14x - 46 = 0$
d) $x^2 + 14x + 46 = 0$

4. If 29x + 37y = 103 and 37x + 29y = 95 then **[1]**

a) x = 3, y = 2
b) x = 2, y = 1
c) x = 2, y = 3
d) x = 1, y = 2

5. For what values of k, the equation $kx^2 - 6x - 2 = 0$ has real roots? **[1]**

a) $k \geq \frac{-9}{2}$
b) None of these

c) $k \leq -2$ d) $k \leq \frac{-9}{2}$

6. If P(-1, 1) is the midpoint of the line segment joining A(-3, b) and B(1, b + 4) then b = ? **[1]**

a) 0 b) 2

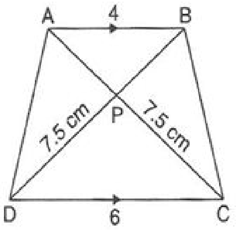
c) 1 d) -1

7. $\triangle ABC \sim \triangle PQR$. If PQ = 3 cm, QR = 2 cm and RP = 2.5 cm, BC = 4 cm, then perimeter of $\triangle ABC$ is **[1]**

a) 20 cm. b) 12 cm.

c) 15 cm. d) 18 cm.

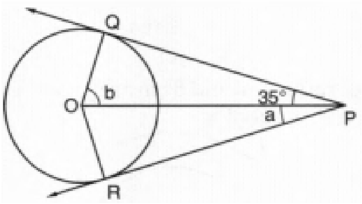
8. In the given figure, if AB||DC, then AP is equal to **[1]**



a) 5 cm. b) 7 cm.

c) 6 cm. d) 5.5 cm.

9. In Figure, PQ and PR are tangents drawn from P to a circle with centre O. If $\angle OPQ = 35^\circ$, then **[1]**



a) a = 40°, b = 50° b) a = 30°, b = 60°

c) a = 45°, b = 45° d) a = 35°, b = 55°

10. (sec A + tan A) (1 – sin A) **[1]**

a) cos A b) sec A

c) sin A d) cosec A

11. If the angles of elevation of a tower from two points at distances 'm' and 'n' where m > n from its foot and in the same line from it are 30° and 60°, then the height of the tower is **[1]**

a) $\sqrt{mn}$ b) $\sqrt{m-n}$

c) $\sqrt{\frac{m}{n}}$ d) $\sqrt{m+n}$

12. If θ is an acute angle such that $\sec^2\theta = 3$, then the value of $\frac{\tan^2\theta - cosec^2\theta}{\tan^2\theta + cosec^2\theta}$ is **[1]**

a) $\frac{1}{7}$ b) $\frac{3}{7}$

c) $\frac{2}{7}$ d) $\frac{4}{7}$

13. A piece of wire 20cm long is bent into the form of an arc of a circle subtending an angle of 60^o at its centre. The radius of the circle is **[1]**

a) $\frac{20}{6+\pi}$ cm b) $\frac{30}{6+\pi}$ cm

c) $\frac{60}{\pi}$ cm d) $\frac{15}{6+\pi}$ cm

14. A chord of a circle of radius 10 cm subtends a right angle at the centre. The area of the minor segments (given, π = 3.14) is **[1]**

a) 32.5 cm^2
b) 34.5 cm^2
c) 30.5 cm^2
d) 28.5 cm^2

15. A ticket is drawn from a bag containing 100 tickets numbered from 1 to 100. The probability of getting a ticket with a number divisible by 10 is **[1]**

a) $\frac{3}{10}$
b) $\frac{1}{10}$
c) $\frac{4}{10}$
d) $\frac{1}{5}$

16. If $\sum f_i u_i$ = 29, $\sum f_i$ = 30, a = 47.5 and h = 15, then the value of $\overline{x}$ is **[1]**

a) 63
b) 26
c) 64
d) 62

17. A sphere of radius 6 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 8 cm. If the sphere is submerged completely, then the surface of the water rises by **[1]**

a) 4.5 cm
b) 4 cm
c) 2 cm
d) 3 cm

18. The wickets taken by a bowler in 10 cricket matches are 2, 6, 4, 5, 0, 3, 1, 3, 2, 3. The mode of the data is **[1]**

a) 1
b) 2
c) 4
d) 3

19. **Assertion (A):** The point (0,-3) lies on the y-axis. **[1]**
Reason (R): The x - coordinate of the point on y-axis is zero.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

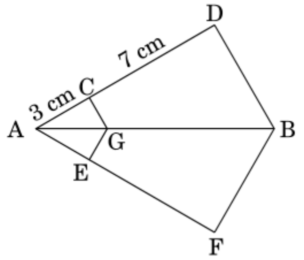
20. **Assertion (A):** If a number x is divided by y(x, y) (both x and y are positive) then remainder will be less than x. **[1]**
Reason (R): Dividend = Divisor $\times$ Quotient + Remainder.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

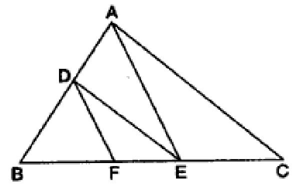
21. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the pair of linear equations intersect at a point, are parallel or coincident: $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$ **[2]**

22. In figure, GC || BD and GE || BF. If AC = 3 cm and CD = 7 cm, then find the value of $\frac{AE}{AF}$ **[2]**

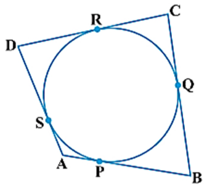


OR

In figure DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$



23. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC **[2]**



24. Prove that: (cosec A – sin A) (sec A – cos A) = $\frac{1}{\tan A+\cot A}$ **[2]**

25. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: **[2]**

   i. minor segment
   ii. major sector.

OR

Four cows are tethered at the four corners of a square field of side 50 m such that each can graze the maximum unshared area. What area will be left ungrazed? [Take π= 3.14.]

Section C

26. Prove that $7\sqrt{5}$ is irrational. **[3]**

27. Find a quadratic polynomial whose sum and product of the zeroes are $-\frac{21}{8}$ and $\frac{5}{16}$ respectively. Also find the zeroes of the polynomial by factorisation. **[3]**

28. Solve the pair of linear equations $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{3}x - \sqrt{8}y = 0$ by substitution method. **[3]**

OR

A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

29. From an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that. **[3]**

   i. PA.PB = PN^2 - AN^2
   ii. PN^2 - AN^2 = OP^2 - OT^2
   iii. PA.PB = PT^2

30. In $\triangle ABC$, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$. Find the value of cos A cos C - sin A sin C **[3]**

OR

Prove the trigonometric identity: $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta} = \tan\theta$

31. A bag contains cards numbered 1 to 49. Find the probability that the number on the drawn card is : **[3]**

   i. an odd number
   ii. a multiple of 5
   iii. Even prime

Section D

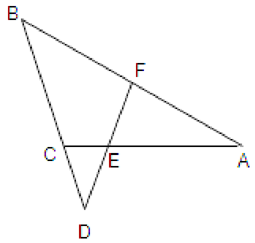
32. Determine whether the given quadratic equation have real roots and if so, find the roots **[5]**

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

OR

A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the number.

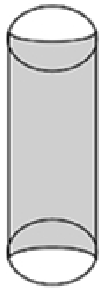
33. In the given figure , a $\angle AEF = \angle AFE$ and E is the mid-point of CA. Prove that $\frac{BD}{CD} = \frac{BF}{CE}$ **[5]**



34. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid (Use π = 22/7) **[5]**

OR

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



35. The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by all the three methods discussed in this section. **[5]**

Percentage of female teachers	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of states/U.T.	6	11	7	4	4	2	1

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) Write first four terms are in AP for the given situations.

(ii) What is the minimum number of days he needs to practice till his goal is achieved?

OR

Out of 41, 30, 37 and 39 which term is not in the AP of the above given situation?

(iii) How many second takes after 5^{th} days?

37. **Read the text carefully and answer the questions:** **[4]**

The Chief Minister of Delhi launched the, ‘Switch Delhi’, an electric vehicle mass awareness campaign in the National Capital. The government has also issued tenders for setting up 100 charging stations across the city. Each station will have five charging points. For demo charging station is set up along a straight line and has charging points at $A\left(\frac{-7}{3}, 0\right)$, $B\left(0, \frac{7}{4}\right)$, C(3, 4), D(7, 7) and E(x, y). Also, the distance between C and E is 10 units.

(i) What is the distance DE?

(ii) What is the value of x + y?

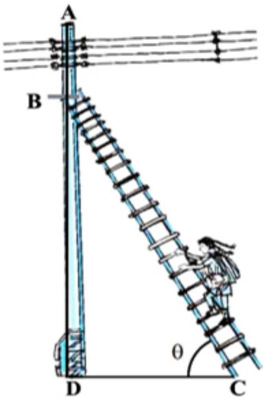
OR

What is the ratio in which B divides AC?

(iii) Points C, D, E are collinear or not?

38. **Read the text carefully and answer the questions:** **[4]**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that $\cos\theta = \frac{\sqrt{3}}{2}$, to reach the required position.



(i) Find the length BD?

(ii) Find the length of ladder.

OR

If the height of pole and distance BD is doubled, then what will be the length of the ladder?

(iii) How far from the foot of the pole should she place the foot of the ladder?

Solution

Section A

1.

(b) coprime

Explanation: We know that the co-prime numbers have no factor in common, or, their HCF is 1.

Thus, p^2 and q^2 have the same factor with exponent 2 each. which again will not have any common factor.

Thus we can conclude that p^2 and q^2 are co-prime numbers.

2.

(b) p^3q^2

Explanation: We know that LCM = product of the highest powers of all the prime factors of the numbers pq^2, p^3q^2

LCM = p^3q^2

3.

(b) $x^2 - 14x + 46 = 0$

Explanation: Given: $\alpha = 7 + \sqrt{3}$ and $\beta = 7 - \sqrt{3}$

$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 (7 + \sqrt{3} + 7 - \sqrt{3})x + (7 + \sqrt{3})(7 - \sqrt{3}) = 0$

$\Rightarrow x^2 - 14x + (49 - 3) = 0$

$\Rightarrow x^2 - 14x + 46 = 0$

4.

(d) x = 1, y = 2

Explanation: 29x + 37y=103 ....(i)

37x+29y=95 ....(ii)

Adding (i) and (ii), we get 66 (x + y) = 198 ⇒ x + y = 3.

Subtracting (ii) from (i), we get 8 (y - x) = 8 ⇒ y - x = 1.

Solve above equations we get

x = 1, y = 2

5. **(a)** $k \geq \frac{-9}{2}$

Explanation: For real roots, we must have, $b^2 - 4ac \geq 0$.

$(-6)^2 - 4 \times k \times (-2) \geq 0 \Rightarrow 36 + 8k \geq 0$

$\Rightarrow 8k \geq -36 \Rightarrow k \geq \frac{-9}{2}$.

6.

(d) -1

Explanation: we have $\frac{b+(b+4)}{2} = 1 \Rightarrow 2b + 4 = 2 \Rightarrow 2b = -2 \Rightarrow b = -1$

7.

(c) 15 cm.

Explanation: Given: $\Delta ABC \sim \Delta PQR$

$\therefore \frac{\text{Perimeter of } \Delta \text{ABC}}{\text{Perimeter of } \Delta \text{PQR}} = \frac{\text{BC}}{\text{QR}}$

$\Rightarrow \frac{\text{Perimeter of } \Delta \text{ABC}}{3+2+2.5} = \frac{4}{2}$

$\Rightarrow$ Perimeter of $\triangle$ABC = 15 cm

8. **(a)** 5 cm.

Explanation: In triangles APB and CPD,

∠APB = ∠CPD [Vertically opposite angles] ∠BAP = ∠ACD [Alternate angles as $AB||CD$]

$\therefore \Delta$APB $\sim \Delta$CPD [AA similarity]

$\therefore \frac{\text{AB}}{\text{CD}} = \frac{\text{CP}}{\text{AP}}$

$\Rightarrow \frac{4}{6} = \frac{AP}{7.5}$

$\Rightarrow$ AP = $\frac{7.5 \times 4}{6}$ = 5 cm

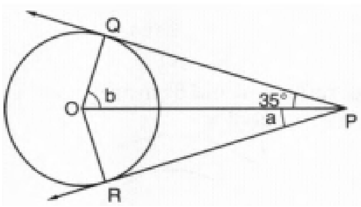
9.

(d) a = 35^o, b = 55^o

Explanation: In the figure, PQ and PR are the tangents drawn from P to the circle with centre O

$\angle OPQ = 35^\circ$

PO is joined



PQ = PR (tangents from P to the circle)

$\angle OPQ = \angle OPR$

$\Rightarrow 35^\circ = \text{a}$

$\Rightarrow a = 35^\circ$

OQ is radius and PQ is tangent $OQ \perp$ PQ

$\Rightarrow \angle OQP = 90^\circ$

In ΔOQP

$\angle \text{POQ} + \angle \text{QPO} = 90^\circ$

$\Rightarrow b + 35^\circ = 90^\circ$

$\Rightarrow \text{b} = 90^\circ - 35^\circ = 55^\circ$

$a = 35^\circ, b = 55^\circ$

10. **(a)** cos A

Explanation: $(\sec A + \tan A)(1 - \sin A)$

$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

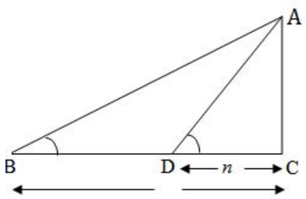
$= [\frac{1+SinA}{CosA}] \times (1 - SinA) = \frac{1-Sin^2A}{CosA}$

$= \frac{\cos^2 A}{\cos A} = \cos A$

Hence, the correct choice is cos A

11. **(a)** $\sqrt{mn}$

Explanation:



In triangle ABC, $\angle$ABC = 30° $\tan 30^\circ = \frac{h}{m}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{m}$

$\Rightarrow h = \frac{m}{\sqrt{3}}$ ....(i)

In triangle ADC, $\angle$ADC = 60°

$\tan 60^\circ = \frac{h}{n}$

$\Rightarrow \sqrt{3} = \frac{h}{n}$

$\Rightarrow h = n\sqrt{3}$ ....(ii)

Multiplying eq.(i) and (ii), we get

$h^2 = \frac{m}{\sqrt{3}} \times n\sqrt{3} = mn$

$\Rightarrow h = \sqrt{mn}$

12. **(a)** $\frac{1}{7}$

Explanation: Given, $\sec^2\theta = 3 \Rightarrow \sec\theta = \frac{\sqrt{3}}{1} = \frac{\text{Hypotenuse}}{\text{Base}}$

By Pythagoras Theorem,

$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$

$(\sqrt{3})^2 = (1)^2 + (Perp.)^2$

$\Rightarrow 3 = 1 + (Perp.)^2 \Rightarrow (Perp.)^2 = 3 - 1 = 2$

$\therefore$ Perpendicular = $\sqrt{2}$

$\therefore \tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$

Now, $\frac{\tan^2\theta - cosec^2\theta}{\tan^2\theta + cosec^2\theta}$

$= \frac{(\sqrt{2})^2 - (\sqrt{\frac{3}{2}})^2}{(\sqrt{2})^2 + (\sqrt{\frac{3}{2}})^2} = \frac{2-\frac{3}{2}}{2+\frac{3}{2}}$

$= \frac{\frac{1}{2}}{\frac{7}{2}} = \frac{1}{2} \times \frac{2}{7} = \frac{1}{7}$

13.

(c) $\frac{60}{\pi}$ cm

Explanation: Given: Length of arc = 20 cm

$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi r = 20$

$\Rightarrow \frac{60^\circ}{360^\circ} \times 2\pi r = 20$

$\Rightarrow \frac{\pi r}{3} = 20$

$\Rightarrow r\left(\frac{\pi}{3}\right) = 20$

$\Rightarrow r\left(\frac{\pi}{3}\right) = 20$

$\Rightarrow r = \frac{60}{\pi}$ cm

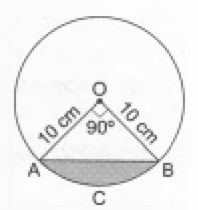
14.

(d) 28.5 cm^2

Explanation:

ar(minor segment A C B A)=ar(sector O A C B O) - ar(ΔOAB)

$= \left(\frac{\pi r^2\theta}{360} - \frac{1}{2} \times r \times r\right)$



$= \left(\frac{3.14 \times 10 \times 10 \times 90}{360} - \frac{1}{2} \times 10 \times 10\right) \text{cm}^2$

$= (78.5 - 50)\text{cm}^2 = 28.5\text{cm}^2$

15.

(b) $\frac{1}{10}$

Explanation: Number of possible outcomes = {10, 20, 30, 40, 50, 60, 70, 80, 90, 100} = 10

Number of Total outcomes = 100

$\therefore$ Required Probability = $\frac{10}{100} = \frac{1}{10}$

16.

(d) 62

Explanation: $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

= $47.5 + \frac{29}{30} \times 15$

= $47.5 + \frac{29}{2}$

= 47.5 + 14.5

= 62

17. **(a)** 4.5 cm

Explanation: Radius of sphere (r) = 6 cm

Volume $= \left(\frac{1}{3}\right)\pi r^3 = \left(\frac{4}{3}\right)\pi(6)^3 \mathrm{cm}^3$

$= \left(\frac{4}{3}\right) \times 216\pi = 4 \times 72\pi \mathrm{cm}^3$ = 28871 cm^3

Radius of vessel (r^2) = 8 cm

Let height of water level = h

$\therefore$ Volume of water raised = $\pi r_2^2 h$

$\therefore \pi r_2^2 h = 288\pi \Rightarrow (8)^2 h = 288$

$\Rightarrow h = \frac{288}{8\times 8} = \frac{36}{8} = \frac{9}{2}\mathrm{cm}$

$\therefore$ Height = 4.5 cm

18.

(d) 3

Explanation: In the given data, the frequency of 3 is more than those other wickets taken by a bowler.

Therefore, Mode of given data is 3.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(d) A is false but R is true.

Explanation: Remainder is less than by divisor not by divident.

Section B

21. Given equations are

$9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing equation $9x + 3y + 12 = 0$ with $a_1x + b_1y + c_1 = 0$

and $18x + 6y + 24 = 0$ with

$a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 9$, $b_1 = 3$, $c_1 = 12$, $a_2 = 18$, $b_2 = 6$, $c_2 = 24$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Hence, lines are coincident.

22. By Basic proportionality theorem which states that if a line is drawn parallel to one side of a triangle the other two sides in distinct points, then the other two sides are divided in the same ratio.

$\frac{AC}{AD} = \frac{AG}{AB}$ ...(i)

Similarly, FE || BF

Therefore, by Basic proportionality theorem

$\frac{AE}{AF} = \frac{AG}{AB}$ ...(ii)

From (i) and (ii), we have

$$\frac{AC}{AD} = \frac{AE}{AF}$$

$$\Rightarrow \frac{3}{10} = \frac{AE}{AF}$$

Hence, the value of $\frac{AE}{AF} = \frac{3}{10}$

OR

In $\triangle ABE$, we have $DF || AE$, then

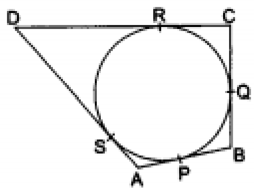
$\frac{BD}{AD} = \frac{BF}{FE}$ [By BPT] ...... (i)

In $\triangle ABC$, we have $DE || AC$, then

$\frac{BD}{AD} = \frac{BE}{EC}$ [By BPT] ...... (2)

From (i) and (2), We get

$\frac{BF}{FE} = \frac{BE}{EC}$

23. 

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

AP = AS, ... (i) [tangents from A]

BP = BQ, ... (ii) [tangents from B]

CR = CQ, ... (iii) [tangents from C]

DR = DS. ... (iv) [tangents from D]

AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS) [using (i), (ii), (iii), (iv)]

= (AS + DS) + (BQ + CQ)

= AD + BC.

Hence, AB + CD = AD + BC.

24. L.H.S. = (cosec A − sin A)(sec A − cos A) = $\left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A} \frac{\sin^2 A}{\cos A}$ = sin A cos A

R.H.S. = $\frac{1}{\tan A+\cot A}$

$= \frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}}$

$= \frac{\cos A \sin A}{\sin^2 A+\cos^2 A}$

= sin A cos A

Hence, L.H.S. = R.H.S.

25. i. r = 10 cm, $\theta = 90^{\circ}$

Area of minor sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times 3.14 \times 10 \times 10 = 78.5 \text{cm}^2$

Area of $\triangle OAB = \frac{OA \times OB}{2}$

$= \frac{10 \times 10}{2} = 50 \text{cm}^2$

∴ Area of the minor segment

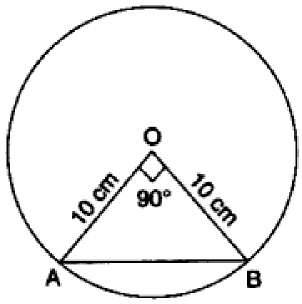
= Area of minor sector - Area of $\triangle OAB$

$= 78.5 \text{cm}^2 - 50 \text{cm}^2 = 28.5 \text{cm}^2$

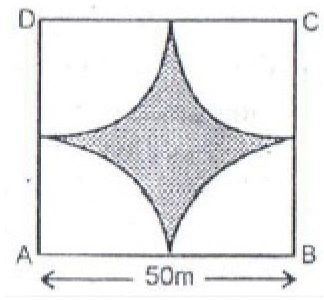
ii. Area of major sector = $\pi x^2 - 28.5$

$= 3.14 \times 10 \times 10 - 28.5$

$= 314 - 28.5 = 285.5 \text{cm}^2$



OR



Shaded area = area of square − 4 (area of sector)

$= \left[(50 \times 50) - \frac{4\times\pi\times(25)^2\times 90}{360}\right] m^2$

$= [2500 - 3.14 \times 25 \times 25] m^2$

$= [2500 - 1962.5] \mathrm{m}^2$

$= 537.5\ \mathrm{m}^2$

Section C

26. We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers *a* and *b* ($b \neq 0$)

such that

$7\sqrt{5} = \frac{a}{b}$

$\Rightarrow \sqrt{5} = \frac{a}{7b}$ ....(1)

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

27. We know, quadratic polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

Given, Sum of zeroes = $-\frac{21}{8}$ and Product of zeroes = $\frac{5}{16}$

$\therefore$ Quadratic Polynomial = $x^2 + \frac{21}{8}x + \frac{5}{16}$

$= \frac{1}{16}(16x^2 + 42x + 5)$

$\Rightarrow$ Quadratic polynomial is $16x^2 + 42x + 5$

Now, we rewrite the polynomial as $16x^2 + 2x + 40x + 5$

$= 2x \cdot (8x+1) + 5 \cdot (8x+1)$

$= (2x+5) \cdot (8x+1)$

Now, for Zeros, $(8x+1) \cdot (2x+5) = 0$

$\Rightarrow x = \frac{-1}{8}, \frac{-5}{2}$

28. The given equations are

$\sqrt{2}x - \sqrt{3}y = 0$ ....(i)

$\sqrt{3}x - \sqrt{8}y = 0$ ....(ii)

From equation (i), we obtain:

$x = \frac{\sqrt{3}y}{\sqrt{2}}$ ...(iii)

Substituting this value in equation (ii), we obtain:

$\sqrt{3}\left(\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$

$\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$

$y\left(\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$

y = 0

Substituting the value of y in equation (iii), we obtain:

x = 0

$\therefore$ x = 0, y = 0

Hence the solution of given equation is (0,0).

OR

Let us suppose that the digit at unit place be x

Suppose the digit at tens place be y.

Thus, the number is 10y + x.

According to question it is given that the number is 4 times the sum of the two digits.

Therefore, we have

10y + x = 4(x + y)

$\Rightarrow$ 10y + x = 4x + 4y

$\Rightarrow$ 4x + 4y - 10y - x = 0

$\Rightarrow$ 3x - 6y = 0

$\Rightarrow$ 3(x - 2y) = 0

$\Rightarrow$ x - 2y = 0

After interchanging the digits, the number becomes 10x + y.

Again according to question If 18 is added to the number, the digits are reversed.

Thus, we have

(10y + x) + 18 = 10x + y

$\Rightarrow$ 10x + y - 10y - x = 18

$\Rightarrow$ 9x - 9y = 18

$\Rightarrow$ 9(x - y) = 18

$\Rightarrow x - y = \frac{18}{9}$

$\Rightarrow$ x - y = 2

Therefore, we have the following systems of equations

x - 2y = 0 ....(1)

x - y = 2...............(2)

Here x and y are unknowns. Now let us solve the above systems of equations for x and y.

Subtracting the equation (1) from the (2), we get

(x - y) - (x - 2y) = 2 - 0

$\Rightarrow$ x - y - x + 2y = 2

$\Rightarrow$ y = 2

Now, substitute the value of y in equation (1), we get

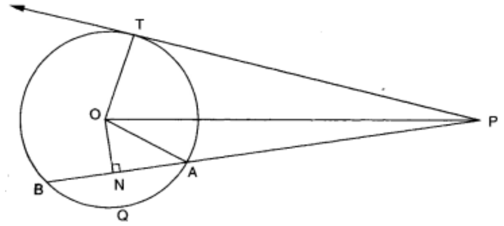
x - 2 $\times$ 2 = 0

$\Rightarrow$ x - 4 = 0

$\Rightarrow$ x = 4

Therefore the number is 10 $\times$ 2 + 4 = 24

Thus the number is 24

29. 

i. PA .PB = (PN - AN)(PN + BN)

= (PN - AN) (PN + AN) $\left[\begin{array}{c}\because ON \perp AB \\ \therefore N \text{ is the mid-point of } AB \\ \Rightarrow AN = BN\end{array}\right]$

= PN^2 - AN^2

ii. Applying Pythagoras theorem in right triangle PNO, we obtain

$OP^2 = ON^2 + PN^2$

$\Rightarrow PN^2 = OP^2 - ON^2$

$\therefore PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$

$= OP^2 - (ON^2 + AN^2)$

$= Op^2 - OA^2$ [Using Pythagoras theorem in ΔONA]

$= OP^2 - OT^2$ $[\because OA = OT = \text{ radius}]$

iii. From (i) and (ii), we obtain

$PA.PB = PN^2 - AN^2$ and $PN^2 - AN^2 = OP^2 - OT^2$

$\Rightarrow PA.PB = OP^2 - OT^2$

Applying Pythagoras theorem in $\triangle OTP$, we obtain

$OP^2 = OT^2 + PT^2$

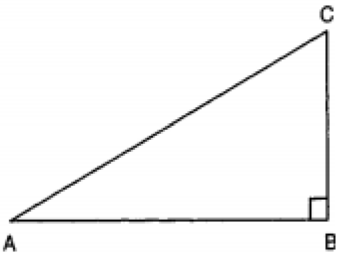
$\Rightarrow OP^2 - OT^2 = PT^2$

Thus, we obtain

PA.PB = OP^2 - OT^2

and OP^2 - OT^2 = PT^2

Hence, PA.PB = PT^2.

30. 

we have,

$\tan A = \frac{1}{\sqrt{3}} = tan 30^\circ$

$\therefore A = 30^\circ$

In $\triangle$ABC, we have

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 30^\circ + 90^\circ + \angle C = 180^\circ$

$\Rightarrow 120^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$

So,

cos A.cos C - sin A.sin C

$= \cos 30^\circ . \cos 60^\circ - \sin 30^\circ . \sin 60^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$

OR

Taking L.H.S. : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$ $[\because sin^2\theta + cos^2\theta = 1]$

$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$

= $\tan\theta$

Hence Proved, LHS = RHS

31. Total cards = 49 so n=49

i. . Odd No. 1, 3, 5, 7, 9,11,13,15,17,19, 21, 23, 25, 27, 29, 31, 33, 35,37,39,41,43,45,47,49= 25 so m=25

P(odd number) = $\frac{m}{n} = \frac{25}{49}$

ii. Multiple of 5 are 5,10,15,20,25,30, 35,40,45 = 9 So m=9

P(multiple of 5) = $\frac{m}{n} = \frac{9}{49}$

iii. The only even prime number is 2 So m=1

P(even prime) = $\frac{m}{n} = \frac{1}{49}$

Section D

32. We have,

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Here, $a = \sqrt{3}, b = 10$ and $c = -8\sqrt{3}$

$\therefore D = b^2 - 4ac$

$= (10)^2 - 4 \times (\sqrt{3}) \times (-8\sqrt{3})$

= $100 + 96 = 196 > 0$

$\therefore D > 0$

So, the given equation has real roots, given by

$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-10 + \sqrt{196}}{2 \times \sqrt{3}}$

$= \frac{-10 + 14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$

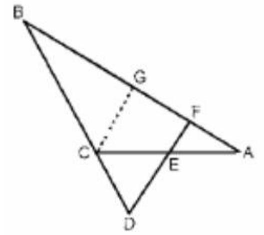
And, $\beta = \frac{-b - \sqrt{D}}{2a}$

$= \frac{-10-\sqrt{196}}{2\sqrt{3}}$
$= \frac{-10-14}{2\sqrt{3}} = -\frac{24}{2\sqrt{3}} = -\frac{12}{\sqrt{3}}$
$= -\frac{4\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = -4\sqrt{3}$
$\therefore\ x = \frac{2}{\sqrt{3}}, -4\sqrt{3}$

OR

Let the ten's digit be x and the one's digit be y.
The number will be 10x + y
Given, a product of digits is 24
$\therefore$ xy = 24
or, y = $\frac{24}{x}$ ...(i)
Given that when 18 is subtracted from the number, the digits interchange their places.
$\therefore$ 10x + y - 18 = 10y + x
or, 9x - 9y = 18
Substituting y from equation (i) in equation (ii), we get
$9x - 9\left(\frac{24}{x}\right)$ = 18
or, $x - \frac{24}{x}$ = 2
or, x^2 - 24 - 2x = 0
or, x^2 - 2x - 24 = 0
or, x^2 - 6x + 4x - 24 = 0
or, x(x - 6) + 4(x - 6) = 0
or, (x - 6)(x + 4) = 0
or, x - 6 = 0 and x + 4 = 0
or, x = 6 and x = -4
Since, the digit cannot be negative, so, x = 6
Substituting x = 6 in equation (i), we get
y = $\frac{24}{6}$ = 4
$\therefore$ The number = 10(6) + 4 = 60 + 4 = 64

33. Given, $\angle AEF = \angle AFE$ and E is the mid-point of CA.
To prove, $\frac{BD}{CD} = \frac{BF}{CE}$
Construction Draw a line CG parallel to DF$(CG||DF)$.



Proof : $\angle AEF = \angle AFE$ and E is the mid-point of CA
$\therefore CE = AE = \frac{AC}{2}$ ...(i)
In $\triangle BDF, CG||DF$
By Basic proportionality theorem,
$\frac{BD}{CD} = \frac{BF}{GF}$ ... (ii)
In ΔAFE,
$\angle AEF = \angle AFE$ [$\because$given]
$\Rightarrow AF = AE$ [[$\because$Since, sides opposite to equal angles are equal]
$\Rightarrow AF = AE = CE$ [[$\because$From Eq(i)] ....(iii)
In ΔACG, E is the midpoint of AC and$EF||CG$,
$\therefore FG = AF$ [$\because AE = CE$] ...(iv)
From Eq(ii), Eq(iii) and Eq(iv),
$\frac{BD}{CD} = \frac{BF}{GF}$

$\frac{BD}{CD} = \frac{BF}{CE}$ [$\because GF = AF = CE$]

Hence proved.

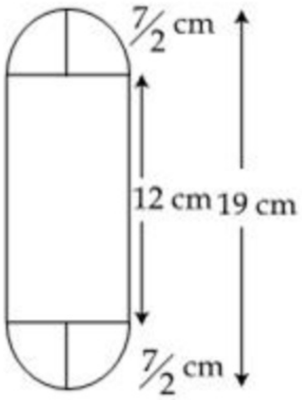
34. Diameter of the cylinder = 7 cm

Therefore radius of the cylinder = $\frac{7}{2} cm$

Total height of the solid = 19 cm

Therefore, Height of the cylinder portion = 19 - 7 = 12 cm

Also, radius of hemisphere = $\frac{7}{2}$ cm



Let V be the volume and S be the surface area of the solid. Then,

V = Volume of the cylinder + Volume of two hemispheres

$\Rightarrow \quad V = \left\{\pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)\right\} \text{cm}^3$

$\Rightarrow \quad V = \pi r^2\left(h + \frac{4r}{3}\right) \text{cm}^3$

$\Rightarrow \quad V = \left\{\frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times \left(12 + \frac{4}{3} \times \frac{7}{2}\right)\right\} \text{cm}^3 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{50}{3} \text{cm}^3 = 641.66 \text{cm}^3$

and,

S = Curved surface area of cylinder + Surface area of two hemispheres

$\Rightarrow \quad S = \left(2\pi rh + 2 \times 2\pi r^2\right) \text{cm}^2$

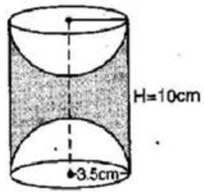
$\Rightarrow \quad S = 2\pi r(h + 2r) \text{cm}^2$

$\Rightarrow \quad S = 2 \times \frac{22}{7} \times \frac{7}{2} \times \left(12 + 2 \times \frac{7}{2}\right) \text{cm}^2$

$= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 19\right) \text{cm}^2$

= 418 cm^2

OR



TSA of the article = $2\pi rh + 2(2\pi r^2)$

= 2π (3.5)(10) + 2[2π $(3.5)^2$]

= $70\pi + 49\pi$

= 119 π

= 119 $\times \frac{22}{7}$

= 374 cm^2

35. Let, $a = 50$

C.I.	Number of states/ U.T. (f_i)	x_i	$d_i = x_i - 50$	$f_i d_i$
15 - 25	6	20	-30	-180
25 - 35	11	30	-20	-220
35 - 45	7	40	-10	-70

45 - 55	4	50	0	0
55 - 65	4	60	10	40
65 - 75	2	70	20	40
75 - 85	1	80	30	30

From table, $\Sigma f_i d_i = -360, \Sigma f_i = 36$

we know that, mean=$\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 50 + \frac{-360}{35}$

$= 39.71$

Section E

36. **Read the text carefully and answer the questions:**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) 51, 49, 47, ... 31 AP

d = -2

First 4 terms of AP are: 51, 49, 47, 45 ...

(ii) 51, 49, 47, ... 31 AP

d = -2

t_n = a +(n - 1)d

31 = 51 + (n - 1) (-2)

31 = 51 - 2n + 2

31 = 53 - 2n

31 - 53 = -2n

-22 = -2n

n = 11

i.e., he acheived his goal in 11 days.

OR

The given AP is

51, 49, 47, 45, 43, 41, 39, 37, 35, 33, 31, 29

$\therefore$ 30 is not in the AP.

(iii) 51, 49, 47, ... 31 AP

d = -2

t_6 = a + (n - 1)d

= 51 + (6 - 1)(-2)

= 51 + (-10)

= 41 sec

37. **Read the text carefully and answer the questions:**

The Chief Minister of Delhi launched the, ‘Switch Delhi’, an electric vehicle mass awareness campaign in the National Capital. The government has also issued tenders for setting up 100 charging stations across the city. Each station will have five charging points. For demo charging station is set up along a straight line and has charging points at $A\left(\frac{-7}{3}, 0\right)$, $B\left(0, \frac{7}{4}\right)$, C(3, 4), D(7, 7) and E(x, y). Also, the distance between C and E is 10 units.

(i) Here, CD = $\sqrt{(7-3)^2+(7-4)^2}$

= $\sqrt{4^2+3^2}$ = 5 units

Also, it is given that CE = 10 units

Thus, DE = CE - CD = 10 - 5 = 5 units ($\because$ A, B, C, E are a line)

(ii) Since, CD = DE = 5 units

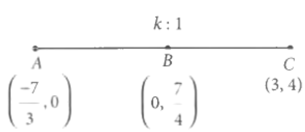
$\therefore$ D is the midpoint of CE.

$\therefore$ $\frac{x+3}{2}$ = 7 and $\frac{y+4}{2}$ = 7

$\Rightarrow$ x = 11 and y = 10 $\Rightarrow$ x + y = 21

OR

Let B divides AC in the ratio k : 1, then



$\frac{7}{4} = \frac{4k+0}{k+1}$

$\Rightarrow$ 7k + 7 = 16 k

$\Rightarrow$ 7 = 9k

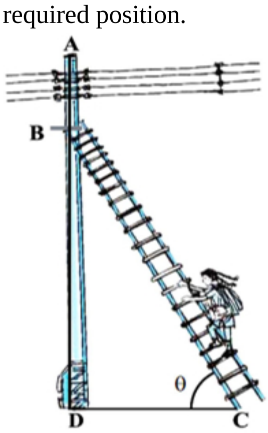
$\Rightarrow$ k = $\frac{7}{9}$

Thus, the required ratio is 7 : 9.

(iii)The points C, D and E are collinear.

38. **Read the text carefully and answer the questions:**

In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle of θ to the horizontal such that cos θ = $\frac{\sqrt{3}}{2}$, to reach the required position.



(i) Length BD = AD - AB = 10 - 2.5 = 8.5

(ii) The length of ladder BC

In $\triangle$BDC

cos $\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow$ θ = 30^o

sin 30^o = $\frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{8.5}{BC}$

$\Rightarrow BC = 2 \times 8.5$ = 17 m

OR

If the height of pole and distance BD is doubled, then the length of the ladder is

sin 30^o = $\frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{17}{BC}$

$\Rightarrow BC = 2 \times 17$ = 34 m

(iii) Distance between foot of ladder and foot of wall CD

In $\triangle$BDC

cos 30^o = $\frac{CD}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CD}{17}$

$\Rightarrow CD = 8.5\sqrt{3}$ m